YO-YO PHYSICS:

AN ENGINEER'S NOTEBOOK

THE ACADEMIC YO-YO

MONOGRAPH II
IN A SERIES

Don Watson

6/2000

Captain Yo

RICHARD P. FEYNMAN, 1918-1988, IN A BRIEF SENTENCE,* GAVE US FIVE KEY WORDS. PRESENTED IN LOGIC FORM THEY ARE:

$$\begin{array}{l} \quad \text{OBSERVATION} \\ + \ \text{REASON} \\ + \ \text{EXPERIMENT} \\ \hline = \ \text{SCIENTIFIC METHOD} \end{array}$$

FOR AN ALWAYS INTERESTING AND OFTEN HILARIOUS ACCOUNT OF FEYNMAN'S LIFE IN AND OUT OF SCIENCE, SEE "SURELY YOU'RE JOKING, MR. FEYNMAN!"; BANTAM BOOKS, 1988.

* * *

* FROM HIS "LECTURES ON PHYSICS", CALIFORNIA INSTITUTE OF TECHNOLOGY, 1963.

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MONOGRAPH II
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PENCILED AND
PUBLISHED BY:

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INTRODUCTION

A WELL-DESIGNED EXPERIMENT WITH COMMON MATERIALS AND GOOD MEASURING EQUIPMENT CAN DETERMINE THE MOMENT OF INERTIA AND RADIUS OF GYRATION OF AN "ACADEMIC YO-YO" WITHOUT GREAT EFFORT. BÜRGER (SEE REFERENCES, 2) DEFINES THAT YO-YO AS HAVING A "STRING OF NO APPRECIABLE THICKNESS" ON A FIXED RADIUS AXLE, ALLOWING THE YO-YO TO FALL WITH UNIFORM ACCELERATION. IN A POPULAR PHYSICS TEXT (REFERENCES, 3), IT IS SUGGESTED THAT GALILEO, WHILE STUDYING FALLING BODIES USING AN INCLINED PLANE, MIGHT WELL HAVE USED THE (ACADEMIC) YO-YO INSTEAD.

THE EXPERIMENT PRESENTED HERE USES TWO POPULAR YO-YOS ADAPTED TEMPORARILY TO THE ACADEMIC YO-YO FORM WITH VERY THIN FILAMENT FOR STRING IN A WIDE STRING GAP. THIS PROVIDES A NEARLY CONSTANT WOUND STRING (i.e. AXLE) RADIUS, ALLOWING THE YO-YO TO FALL AT VERY NEAR CONSTANT ACCELERATION ALONG THE WOUND FILAMENT. ANALYSIS OF FREE BODY DIAGRAM VECTOR FORCES MATHEMATICALLY RELATES THE MOMENT OF INERTIA AND RADIUS OF GYRATION TO YO-YO WEIGHT, AXLE RADIUS, LENGTH AND TIME OF FALL UNWINDING THE FILAMENT.

MONOGRAPH I IN THIS SERIES PRESENTED AN AREA ELEMENT ANALYSIS METHOD TO DETERMINE THE SAME RESULTS. WHY THEN IS A SECOND

METHOD OF ANY INTEREST? THE EXPERIMENT REQUIRES MUCH LESS EFFORT AND TIME; MORE IMPORTANTLY, IT PROVIDES AN ENTIRELY INDEPENDENT CHECK ON RESULTS OF THE AREA ELEMENT ANALYSIS. GOOD AGREEMENT IN RESULTS FROM THE TWO METHODS BREEDS CONFIDENCE IN THE RESULTS AND THE METHODS. THE EXPERIMENT MIGHT THEN BE THE METHOD OF CHOICE ONCE A YO-YO REACHES AT LEAST PROTOTYPE STATUS.

THE EXPERIMENT PRESENTED HERE OFFERS A VALUABLE LEARNING EXPERIENCE IN EXPERIMENT DESIGN AND EXECUTION. PHYSICS AND MATH TO BE APPLIED, MATERIALS AND EQUIPMENT NEEDED, PROCEDURE, CALCULATIONS, AND EVALUATION OF ERROR SOURCES ARE WELL DEFINED AND ORGANIZED. IN THE CALCULATIONS, ALL UNITS OF MEASURE ARE JUSTIFIED, AND ENGINEERING NOTATION (POWERS OF 10) IS USED TO AVOID RECORDING SIGNIFICANT FIGURES BEYOND THE THIRD DECIMAL PLACE.

THE AUTHOR OFFERS APPRECIATION ONCE MORE TO BRAD COUNTRYMAN AND TOM KUHN, AND TO DALE OLIVER, FOR THEIR PERMISSION TO USE NON-PROPRIETARY INFORMATION ON THEIR RESPECTIVE PRODUCTS IN THIS PUBLICATION. THE NO JIVE 3-IN-1 IS A WHAT'S NEXT INC. PRODUCT; THE TORNADO 2 IS A SPINTASTICS INC. PRODUCT. EXPERIMENT AND AREA ELEMENT ANALYSIS INFORMATION FOR BOTH YO-YOS IS PRESENTED ALLOWING DIRECT COMPARISON OF RESULTS.

THE ACADEMIC YO-YO

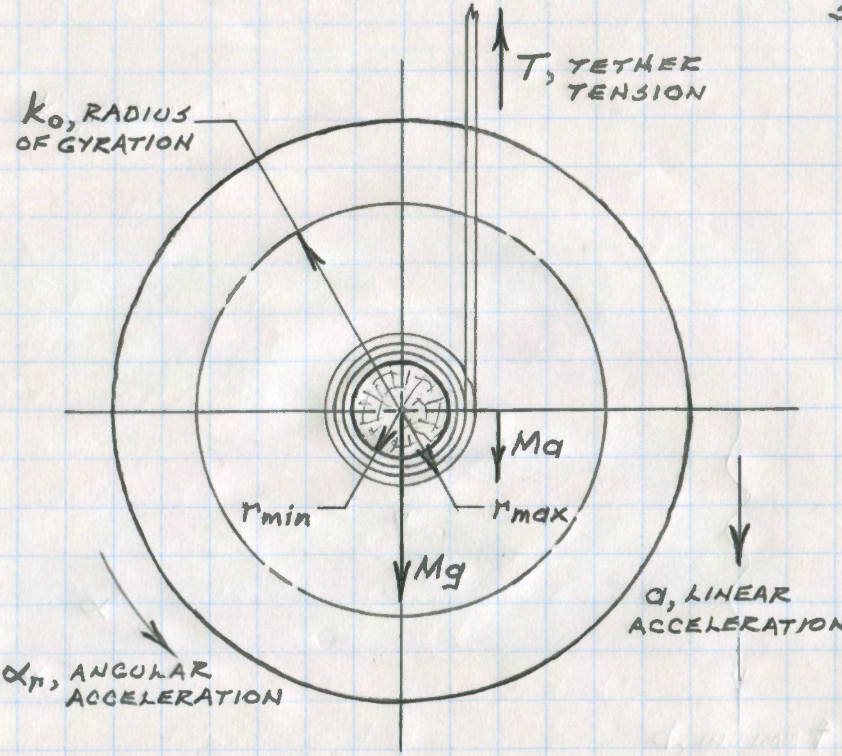
ACADEMIC YO-YO DEFINED

THE YO-YO SUITED FOR ACADEMIC STUDY HAS TWO PRACTICAL AND RELATED REQUIREMENTS: A SIGNIFICANTLY WIDER THAN NORMAL STRING GAP, AND A SLENDER TETHER TO REPLACE THE NORMAL STRING. THESE MODIFICATIONS ALLOW USE OF A LONG TETHER, WHILE LIMITING THE EFFECTIVE AXLE RADIUS r AND ITS LIMITS (r_{min} AND r_{max}); SEE THE ILLUSTRATION ON THE FACING PAGE. EXPERIMENTALLY, THE YO-YO OF WEIGHT M AND AXLE RADIUS r FALLS – UNWINDING THE LAYERED TETHER – A MEASURED DISTANCE d WHILE THE TIME OF FALL t IS MEASURED. GIVEN THE AXLE RADIUS r AS EFFECTIVELY CONSTANT, THE TIME t, AND WITH THE OTHER REQUIRED EXPERIMENTAL VALUES (g, d, AND M) KNOWN, THE MOMENT OF INERTIA I AND THE RADIUS OF GYRATION k_o CAN BE ACCURATELY ESTIMATED WHERE:

$$I = \left(\frac{gt^2}{2d} - 1\right) \cdot Mr^2 \ \text{kg-m}^2$$

$$k_o = \left(\frac{I}{M}\right)^{\frac{1}{2}} \ \text{m}$$

SEE PAGE 6 FOR A DETAILED DERIVATION OF THESE RELATIONSHIPS.



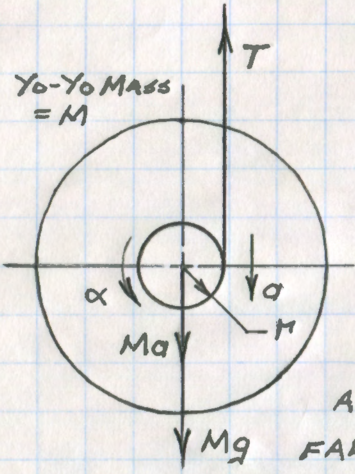
THE ACADEMIC YO-YO

REQUIRED EXPERIMENTAL VALUES:

g = ACCELERATION DUE TO GRAVITY, $9.81\ m/sec^2$
d = LENGTH OF FALL, UNWINDING THE TETHER, m
M = TOTAL YO-YO WEIGHT (MASS), gm
r = EFFECTIVE AXLE RADIUS, $(r_{min}+r_{max})/2$ m
t = TIME OF FALL, UNWINDING THE TETHER, sec

NOTE: MOMENT OF INERTIA I AND RADIUS OF GYRATION k_o ARE CALCULATED FROM THESE VALUES; SPECIFIC VALUES FOR FACTORS T, α_r, AND a ARE NOT NEEDED.

ACADEMIC YO-YO
FREE BODY ANALYSIS



THE YO-YO HERE IS TAKEN TO FALL BY GRAVITY ALONE, UNWINDING THE SLENDER TETHER ALLOWING AXLE RADIUS r TO BE CONSIDERED CONSTANT. THE VECTOR SUM OF FORCES ACTING ON THIS FREELY-FALLING BODY MUST EQUAL THE MASS M MULTIPLIED BY ITS VERTICAL ACCELERATION a, FROM NEWTON'S SECOND LAW. HERE, THE GRAVITY FORCE VECTOR Mg IS OPPOSED BY A TETHER TENSION FORCE VECTOR T. THE VECTOR SUM OF THESE TWO FORCES PRODUCES Ma, THE DOWNWARD FORCE ACCELERATING THE YO-YO. THUS:

$$\sum \text{FORCES} = T + (-Mg) = -Ma$$

$$T = M(g - a)$$

THE LINEAR a AND ANGULAR α ACCELERATIONS ARE DIRECTLY RELATED AS $a = \alpha r$, OR $\alpha = a/r$. IN ROTATING MOTION, NEWTON'S SECOND LAW PROVIDES THAT THE SUMMATION OF TORQUES MUST EQUAL THE PRODUCT OF

THE MOMENT OF INERTIA I AND THE ANGULAR ANGULAR ACCELERATION α; HERE, WITH THE TETHER TENSION T AT RADIUS r:

$$\Sigma \text{TORQUE} = Tr = I\alpha = I\left(\frac{a}{r}\right)$$

$$T = I\left(\frac{a}{r^2}\right)$$

EARLIER, $T = M(g-a)$

ELIMINATING T AND SIMPLIFYING:

$$a = g\left[\frac{1}{1+(I/Mr^2)}\right]$$

THE DISTANCE d THE YO-YO WILL FALL AT ITS ACCELERATION a IN ANY TIME t IS:

$$d = \frac{1}{2}at^2 \quad \text{OR} \quad a = \frac{2d}{t^2}$$

THEN $\dfrac{2d}{t^2} = g\left[\dfrac{1}{1+(I/Mr^2)}\right]$

$$1+(I/Mr^2) = \frac{gt^2}{2d}$$

$$\underline{\underline{I = \left(\frac{gt^2}{2d} - 1\right)Mr^2}} \text{ kg-m}^2$$

AND $\dfrac{gt^2}{2d} = \dfrac{I}{Mr^2} + 1$

$$\underline{\underline{t = \left[\frac{2d}{g}\left(\frac{I}{Mr^2}+1\right)\right]^{\frac{1}{2}}}} \text{ sec}$$

UNITS OF MEASUREMENT

IN THE DERIVED RELATIONSHIPS FOR I AND t ON THE PRECEDING PAGE, THE JUSTIFICATION OF UNITS IS HELPFUL IN ASSURING VALID RESULTS. REWRITING THOSE RELATIONSHIPS, SHOWING ALL UNITS AND THEN CANCELLING THEM EVERYWHERE POSSIBLE, MUST LEAVE THE CORRECTLY DEFINED UNITS FOR THE RESULTS:

$$I = \left(\frac{g \frac{\cancel{m}}{\cancel{sec^2}} \cdot t^2 \cancel{sec^2}}{2d\,\cancel{m}} - 1 \right) \cdot M\,kg \cdot r^2 m^2 = I\ kg\text{-}m^2$$

$$t = \left[\frac{2d\,\cancel{m}}{g \frac{\cancel{m}}{sec^2}} \cdot \left(\frac{I\,\cancel{kg\text{-}m^2}}{M\,\cancel{kg} \cdot r^2 \cancel{m^2}} + 1 \right) \right]^{\frac{1}{2}} = t\ sec$$

* * *

ACADEMIC YO-YO EXPERIMENT

FEYNMAN (INSIDE FRONT COVER) GUIDES US HERE: THE RELATIONSHIPS FOR I AND T PROVIDE FOR "REASON" NEEDED TO PROCEED WITH AN "EXPERIMENT" DESIGN TO YIELD SUFFICIENTLY ACCURATE "OBSERVATION" DEFINING A YO-YO ADAPTED FOR "ACADEMIC" ANALYSIS.

ADAPTATION OF AN EXISTING YO-YO REQUIRES THAT ITS TETHER UNWIND WITH AN EFFECTIVELY CONSTANT RADIUS WHILE FALLING A KNOWN VERTICAL DISTANCE IN A MEASURED TIME. THE "EFFECTIVELY CONSTANT", OR AVERAGE, RADIUS CAN BE ACHIEVED USING A VERY THIN FILAMENT TETHER MATERIAL, LEVEL-WOUND THROUGH A FEW UNIFORM LAYERS IN A TEMPORARILY WIDE STRING GAP.

IN THE EXPERIMENT, ACCELERATION DUE TO GRAVITY ($9.81 m/sec^2$), THE DISTANCE OF VERTICAL FALL, THE WEIGHT OF THE YO-YO, AND THE EFFECTIVE (AVERAGE) AXLE RADIUS MUST BE DETERMINED AS BEGINNING DATA. TRIALS ARE THEN PERFORMED, MEASURING ACCURATELY THE TIME OF FALL. FINALLY THESE VALUES ARE USED TO CALCULATE, WITH SOME CONFIDENCE, THE MOMENT OF INERTIA AND RADIUS OF GYRATION FOR THE YO-YO AT HAND; IN THIS CASE, TOM KUHN'S "NO JIVE 3-IN 1" AND DALE OLIVER'S "TORNADO 2".

EQUIPMENT AND MATERIAL

THE TEST PROCEDURE REQUIRES THE FOLLOWING ITEMS IN THE DETERMINATION OF MOMENT OF INERTIA AND RADIUS OF GYRATION FOR TEST YO-YOS:

1. A "TAKE-APART" TEST YO-YO ASSEMBLY WITH A TEMPORARY AXLE SLEEVE PROVIDING A STRING GAP WIDTH OF AT LEAST 1/4".
2. BIRCH DOWEL OF CORRECT DIAMETER AND LENGTH FOR THE AXLE SLEEVE.
3. NYLON FILAMENT, 0.010" D., 10' LENGTH (FISHING TACKLE LEADER, 8 lb. TEST).
4. A PLANT HOOK OR OTHER MEANS TO SUSPEND THE YO-YO ABOUT 8' ABOVE THE FLOOR.
5. DIGITAL GRAM SCALE, 100gm CAPACITY, 0.10 gm ACCURACY.
6. TAPE MEASURE, 10'.
7. DIAL-INDICATING CALIPER, 1" PER DIAL REVOLUTION, 0.010" GRADUATIONS.
8. DIGITAL STOP-WATCH, 0.01 SEC. ACCURACY.

NOTE: IN ASSEMBLING THE TEST YO-YO WITH THE WIDER STRING GAP, TAKE CARE THAT THE AXLE SCREW WILL HAVE EQUAL AND ADEQUATE THREAD CAPTURE AT BOTH ENDS. IF NOT, USE A SOLID DOWEL AXLE FIRMLY FITTED AT BOTH ENDS OR A LONGER AXLE SCREW.

TEST PROCEDURE

1. RECORD THE TOTAL WEIGHT M OF THE TEST YO-YO TO THE NEAREST 0.10 GM.

2. CAPTURE A FILAMENT END IN A YO-YO HALF WITH THE AXLE SLEEVE; MOUNT THE OTHER HALF TO PROVIDE A STRING GAP AT LEAST 1/4" WIDE.

3. TIE A SMALL FIXED LOOP AT THE HOOK TO SUSPEND THE YO-YO WITHIN 4" OF THE FLOOR; RECORD THE DISTANCE d BETWEEN AXLE CENTER POSITIONS A AND B (ROUGHLY 90").

4. REMOVE THE FILAMENT FROM THE HOOK; RECORD Dmin, THE AXLE SLEEVE DIAMETER.

5. "LEVEL WIND" THE FILAMENT ON THE SLEEVE WITH THREE OR MORE LAYERS OF CLOSELY PACKED TURNS; RANDOM WINDING WITH MANY TURNS CROSSED MUST BE AVOIDED.

6. RECORD Dmax FOR THE WOUND FILAMENT; ADJUST THE VALUE FOR ANY PARTIAL LAYER.

7. CALCULATE AND RECORD THE EFFECTIVE AXLE RADIUS. $r = (D_{max} + D_{min})/4$.

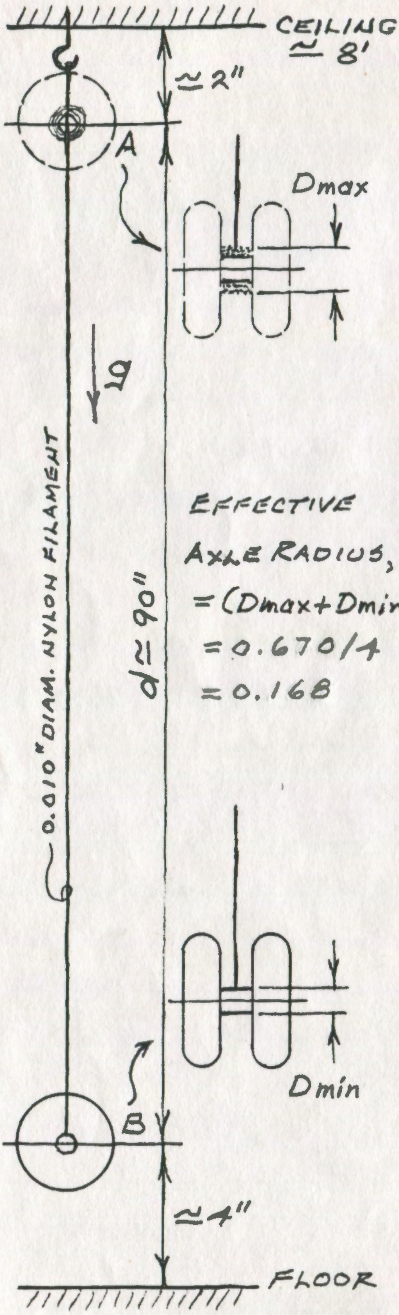
8. SLIP THE FILAMENT (FULLY WOUND) LOOP ON THE HOOK. SIMULTANEOUSLY, RELEASE THE YO-YO AT POINT A AND START THE WATCH; STOP THE WATCH THE INSTANT THE YO-YO HITS POINT B. MAKE SEVEN TRIALS; STRIKE THE HIGHEST AND LOWEST TIMES.

9. CALCULATE t, THE AVERAGE OF FIVE VALUES.

10. TRANSFER THE DATA TO THE CALCULATION WORKSHEET; CALCULATE α AND k_o AS INDICATED.

NAME DW

Yo-Yo NO JIVE 3-IN-1



EFFECTIVE AXLE RADIUS, r
= (Dmax+Dmin)/4
= 0.670/4
= 0.168

TRIAL DATA:

$g = 9.81 \text{ m/sec}^2$

$d =$ 89.5 in

$M =$ 53.4 gm

$D_{max} =$ 0.370 in

$D_{min} =$ 0.300 in

$r =$ 0.168 in

TRIAL TIME:

1. 2.94
2. 2.98
3. 3.00
4. 2.91
5. ~~2.85~~ LOW
6. 3.00
7. ~~3.02~~ HIGH

TOTAL = 14.73 sec

t = TOTAL/5

t = 2.95 sec

SHEET 1 OF 2

CALCULATION WORKSHEET

UNIT CONVERSIONS:

$g = 9.81 \ m/sec^2$

$t = \underline{2.95}$ sec

$d = \underline{89.5}$ in$\cdot 25.4\times10^{-3}$ m/in $= \underline{2.27}$ m

$M = \underline{53.4}$ gm$\times10^{-3}$ kg/gm $= \underline{53.4}\times10^{-3}$ kg

$r = \underline{0.168}$ in$\cdot 25.4\times10^{-3}$ m/in $= \underline{4.27}\times10^{-3}$ m

MOMENT OF INERTIA, J kg-m²:

$$J = \left(\frac{gt^2}{2d} - 1\right) M r^2 \ \text{kg-m}^2$$

$$J = \left(\frac{9.81 \cancel{\frac{m}{sec^2}} \cdot 2.95^2 \cancel{sec^2}}{2 \cdot 2.27 \cancel{m}} - 1\right) \cdot 53.4\times10^{-3}\,kg \cdot (4.27\times10^{-3}\,m)^2$$

$$\underline{\underline{J = 17335 \times 10^{-9}}} \ \text{kg-m}^2$$

RADIUS OF GYRATION, k_o in:

$$k_o = \left(\frac{J}{M}\right)^{\frac{1}{2}} m$$

$$k_o = \left(\frac{17335 \times10^{-9} \cancel{kg}\text{-}m^2}{53.4 \times10^{-3} \cancel{kg}}\right)^{\frac{1}{2}} \Big/ 25.4\times10^{-3} \frac{\cancel{m}}{in}$$

$$\underline{\underline{k_o = 0.71}} \ \text{in}$$

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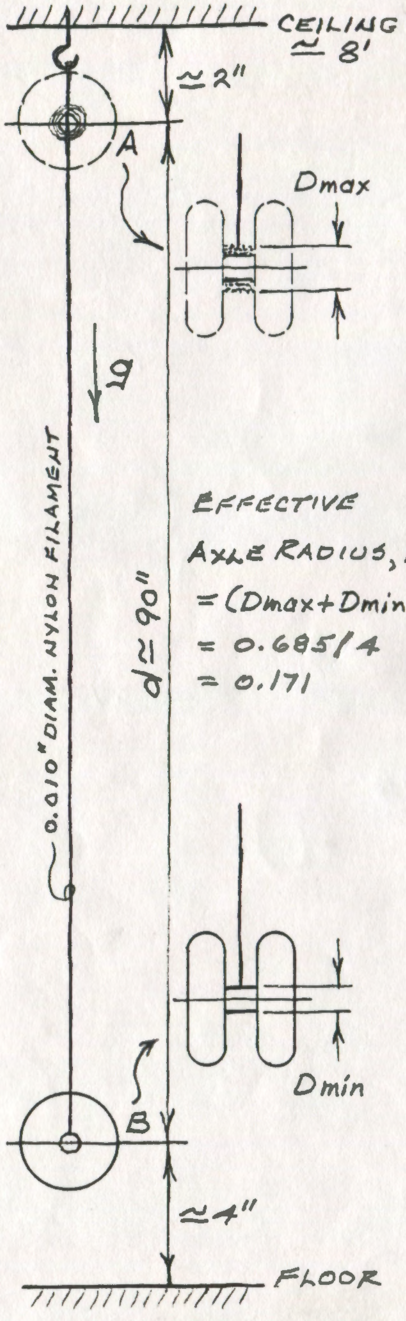
SHEET 2 OF 2

NAME DW

YO-YO TORNADO 2
(WITH RINGS = 8.2 gm)



EFFECTIVE
AXLE RADIUS, r
$= (Dmax + Dmin)/4$
$= 0.685/4$
$= 0.171$

TRIAL DATA:

$g = 9.81\ m/sec^2$
$d =$ 86.5 in
$M =$ 59.5 gm
$Dmax =$ 0.375 in
$Dmin =$ 0.310 in
$r =$ 0.171 in

TRIAL TIME:

1. 3.21
2. ~~3.27~~ HIGH
3. 3.18
4. 3.18
5. ~~3.16~~ LOW
6. 3.19
7. 3.21

TOTAL = 15.97 sec
t = TOTAL/5

$t = 3.19$ sec

SHEET 1 OF 2

CALCULATION WORKSHEET

UNIT CONVERSIONS:

$g = 9.81 \ m/sec^2$

$t = \underline{3.19} \ sec$

$d = \underline{86.6} \ in \cdot 25.4 \times 10^{-3} m/in = \underline{2.20} \ m$

$M = \underline{59.5} \ gm \times 10^{-3} \ kg/gm = \underline{59.5} \times 10^{-3} kg$

$r = \underline{0.171} \ in \cdot 25.4 \times 10^{-3} m/in = \underline{4.34} \times 10^{-3} m$

MOMENT OF INERTIA, I kg-m²:

$$I = \left(\frac{gt^2}{2d} - 1\right) M r^2 \ kg\text{-}m^2$$

$$I = \left(\frac{9.81 \frac{\cancel{m}}{\cancel{sec^2}} \cdot 3.19^2 \cancel{sec^2}}{2 \cdot 2.20 \cancel{m}} - 1\right) \cdot 59.5 \times 10^{-3} kg \cdot (4.34 \times 10^{-3} m)^2$$

$$\underline{\underline{I = 24306 \times 10^{-9}}} \ kg\text{-}m^2$$

RADIUS OF GYRATION, k_o in:

$$k_o = \left(\frac{I}{M}\right)^{\frac{1}{2}} m$$

$$k_o = \left(\frac{24306 \times 10^{-9} \cancel{kg}\text{-}m^2}{59.5 \times 10^{-3} \cancel{kg}}\right)^{\frac{1}{2}} / 25.4 \times 10^{-3} \frac{\cancel{m}}{in}$$

$$\underline{\underline{k_o = 0.80}} \ in$$

DATE 5/6/00 BY DW

GARBAGE IN, GARBAGE OUT?

ACADEMIC YO-YO METHODS USED HERE CANNOT YIELD PRECISION RESULTS; IT'S NECESSARY TO DEFINE LIMITS OF ATTAINABLE ACCURACY.

ASSIGNING REASONABLE LIMITS OF ACCURACY TO EACH MEASURED VARIABLE ($g = 9.81\ m/sec^2$, A KNOWN CONSTANT) FOR A YO-YO "SIMILAR" TO THOSE JUST TESTED:

$t = 3.00 \pm 1\% = 3.00 \pm 0.03$ sec (AVG. 5 TRIALS)

$d = 90.0 \pm 0.3\% = 90.0 \pm 0.25\ in = 2.29 \pm 0.01\ m$

$M = 55.0 \pm 0.2\% = (55.0 \pm 0.10) \times 10^{-3}\ kg$

$r = 0.170 \pm 2\% = (4.32 \pm 0.09) \times 10^{-3}\ m$

J, J_{MAX}, J_{MIN} ARE CALCULATED USING THE BASE VALUES, THEN THE UPPER AND LOWER LIMITS:

$$J \simeq 18800,\ J_{MAX} \simeq 20100,\ J_{MIN} \simeq 17500 \times 10^{-9}\ kg\text{-}m^2$$

$$J \simeq (18800 \pm 7\%) \times 10^{-9}\ kg\text{-}m^2$$

$$k_o \simeq 0.73,\ k_{oMAX} \simeq 0.75,\ k_{oMIN} \simeq 0.71\ in$$

$$k_o \simeq 0.73 \pm 3\%\ in$$

THESE ARE AT LEAST ACCEPTABLE ACCURACIES. SEE "EXPERIMENT TECHNIQUE", PAGE 34, FOR TIPS ON ACHIEVING GOOD ACCURACY.

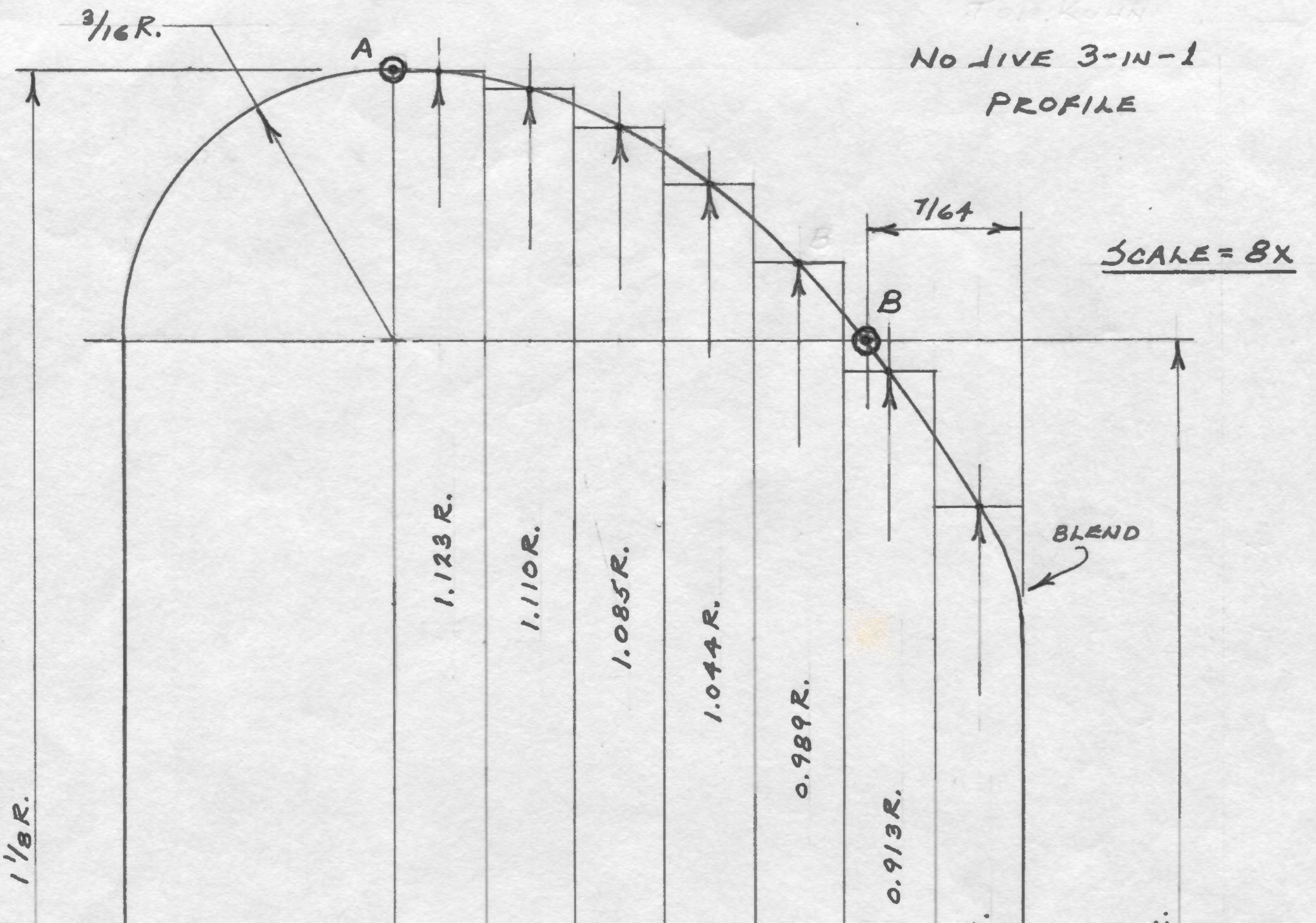
NO JIVE 3-IN-1™
*AREA ELEMENT ANALYSIS

THE FOLLOWING CENTERPIECE SHOWS A LARGE SCALE (8X) SECTION OF THE YO-YO PROFILE. KEY MEASUREMENTS (RADIUS TO Ⓐ, BORE DETAIL, ETC.) WERE TAKEN DIRECTLY FROM THE YO-YO, AS WERE THOSE LOCATING A POINT Ⓑ ON THE OUTER EDGE. A PARABOLIC FRENCH CURVE WAS THEN USED TO DRAW THE CURVE HORIZONTAL AT Ⓐ, PASSING THROUGH Ⓑ, WITH A SLIGHT BLEND AT THE OUTER VERTICAL EDGE. THE 7/16" WIDE PORTION WAS THEN PARTITIONED INTO SEVEN 1/16" SLICES, WITH EACH SLICE RADIUS (℄ TO PARABOLA) LENGTH DIVIDED BY 8 AND RECORDED.

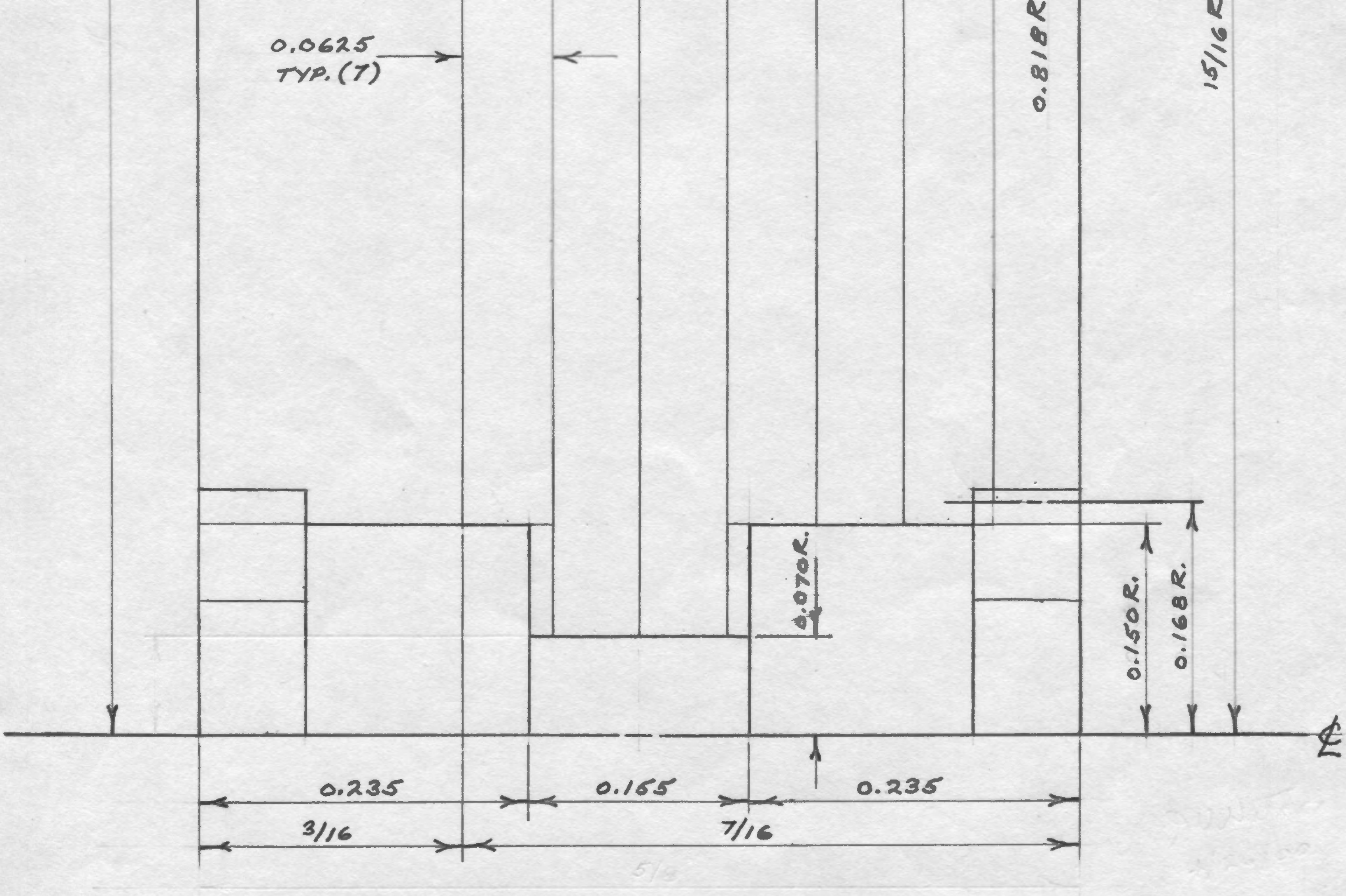
DIMENSIONS FOR EACH OF THE SEVEN SLICES AND THE OTHER TWO AREA ELEMENTS WITH A KNOWN (MAPLE) DENSITY COMPLETE THE DATA NEEDED TO DETERMINE THE WEIGHT M AND MOMENT OF INERTIA J FOR EACH AREA ELEMENT, ① THROUGH ⑨; SEE PAGE 20.

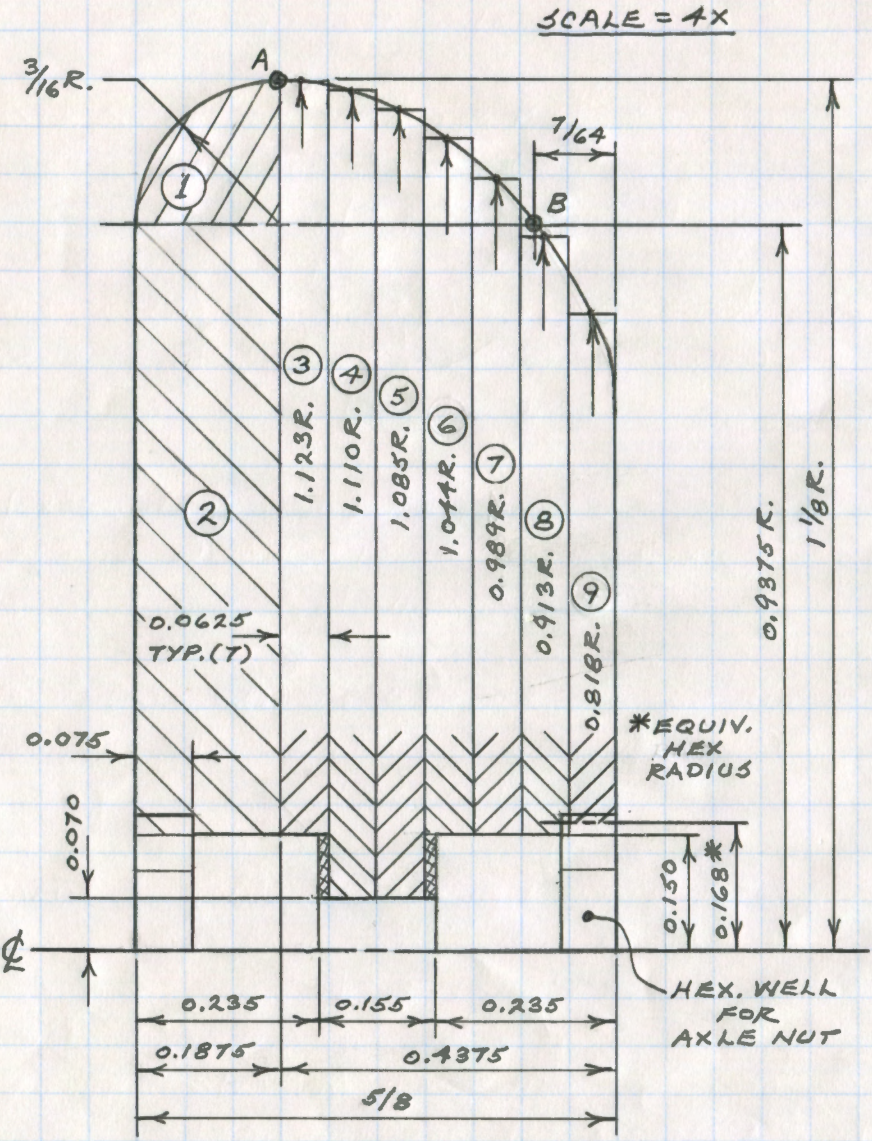
THE PARABOLA SLICE 8X SCALE RADII DIVIDED BY 8 MINIMIZE LINE-WIDTH AND SIGHT ERRORS; ERRORS NOT SO MINIMIZED WHEN MEASURING AT 1X SCALE.

*SEE MONOGRAPH I, "RADIUS OF GYRATION" IN THIS SERIES FOR DETAILED INFORMATION.



NO DIVE 3-IN-1
PROFILE
SCALE = 8X
3/16R.
A
B
7/64
BLEND
1 1/8R.
1.123R.
1.110R.
1.085R.
1.044R.
0.989R.
0.913R.


0.0625
TYP. (7)
0.818R
15/16R
0.070R.
0.150R.
0.168R.
℄
0.235
0.165
0.235
3/16
7/16



NO DIVE 3-IN-1™
PROFILE

YO-YO HALF PROFILE

YO-YO ELEMENT		VOLUME (in^3)
①	$V = \left(\frac{\pi x^2}{4}\right)\cdot 2\pi\cdot(r+0.424x)$	
	$= \frac{\pi\cdot 0.1875^2}{4}\cdot 2\pi\cdot(0.9875+(0.424\cdot 0.1875)) =$	0.176
②	$V^{*} = (x\cdot y)\cdot 2\pi\cdot(r+\frac{y}{2})$	
	$=(0.1875\cdot 0.7875)\cdot 2\pi\cdot(0.150+\frac{0.7875}{2}) =$	0.504
③	$V^{*}=(0.0625\cdot 0.973)\cdot 2\pi\cdot(0.150+\frac{0.973}{2}) =$	0.243
④	$V^{*}=(0.0625\cdot 1.040)\cdot 2\pi\cdot(0.070+\frac{1.040}{2}) =$	0.241
⑤	$V^{*}=(0.0625\cdot 1.015)\cdot 2\pi\cdot(0.070+\frac{1.015}{2}) =$	0.230
⑥	$V^{*}=(0.0625\cdot 0.894)\cdot 2\pi\cdot(0.150+\frac{0.894}{2}) =$	0.210
⑦	$V^{*}=(0.0625\cdot 0.839)\cdot 2\pi\cdot(0.150+\frac{0.839}{2}) =$	0.188
⑧	$V^{*}=(0.0625\cdot 0.763)\cdot 2\pi\cdot(0.150+\frac{0.763}{2}) =$	0.159
⑨	$V^{*}=(0.0625\cdot 0.650)\cdot 2\pi\cdot(0.168+\frac{0.650}{2}) =$	0.126

* = COMMON FORMULA

YO-YO HALF, $V = 2.077\ in^3$

NOTE: THESE HALVES EACH WEIGH 24.8 gm. THUS

$\rho = 24.8/2.077$ $\qquad \rho = 11.94\ gm/in^3$

YO-YO HALF PROFILE

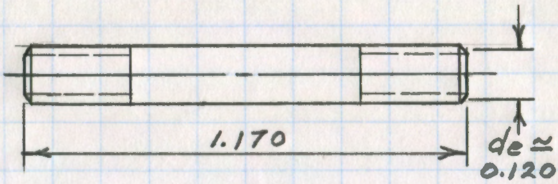
YO-YO ELEMENT	MOMENT OF INERTIA	J (kg-$m^2 \times 10^{-9}$)
(1)	$J = Mk_o^2 = \rho V (r + 0.424x)^2$ $= 11.94 \cdot 0.176 \times 10^{-3} \cdot ((0.9375 + (0.424 \cdot 0.1875)) \cdot 25.4 \times 10^{-3})^2$ $J = 1402 \times 10^{-9}\ kg\text{-}m^2$	1402
(2)	$J^* = \frac{1}{2} M (r_1^2 + r_2^2) = \frac{1}{2} \rho V (r_1^2 + r_2^2)$ $= \frac{11.94 \cdot 0.504 \times 10^{-3}}{2} \cdot (0.150^2 + 0.9375^2) \cdot (25.4 \times 10^{-3})^2 =$	1750
(3)	$J^* = \frac{11.94 \cdot 0.243 \times 10^{-3}}{2} \cdot (0.150^2 + 1.123^2) \cdot (25.4 \times 10^{-3})^2 =$	1201
(4)	$J^* = \frac{11.94 \cdot 0.241 \times 10^{-3}}{2} \cdot (0.070^2 + 1.110^2) \cdot (25.4 \times 10^{-3})^2 =$	1148
(5)	$J^* = \frac{11.94 \cdot 0.230 \times 10^{-3}}{2} \cdot (0.070^2 + 1.085^2) \cdot (25.4 \times 10^{-3})^2 =$	1047
(6)	$J^* = \frac{11.94 \cdot 0.210 \times 10^{-3}}{2} \cdot (0.150^2 + 1.044^2) \cdot (25.4 \times 10^{-3})^2 =$	900
(7)	$J^* = \frac{11.94 \cdot 0.188 \times 10^{-3}}{2} \cdot (0.150^2 + 0.989^2) \cdot (25.4 \times 10^{-3})^2 =$	725
(8)	$J^* = \frac{11.94 \cdot 0.159 \times 10^{-3}}{2} \cdot (0.150^2 + 0.913^2) \cdot (25.4 \times 10^{-3})^2 =$	524
(9)	$J^* = \frac{11.94 \cdot 0.126 \times 10^{-3}}{2} \cdot (0.168^2 + 0.818^2) \cdot (25.4 \times 10^{-3})^2 =$	338

YO-YO HALF, $\underline{\underline{J = 9035}}$ $\times 10^{-9}\ kg\text{-}m^2$

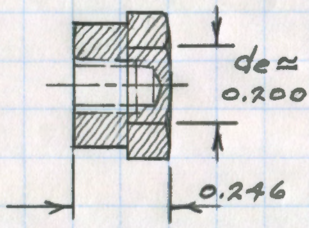
* = COMMON FORMULA

ASSEMBLY PARTS

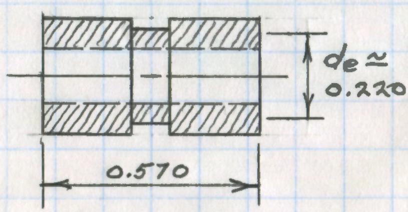
AXLE SCREW (1):
STEEL, 0.138 D.
$M = 1.90\,gm$



HEX NUTS (2);
ALUMINUM
$M = 0.80\,gm$ EACH



AXLE SLEEVE (1):
BIRCH, 0.290 D.
$M = 0.30\,gm$



MOMENT OF INERTIA

I
($\times 10^{-9}\,kg\text{-}m^2$)

AXLE SCREW: $M = 1.90\,gm,\ r \simeq \frac{de}{2} = 0.060\,in$

$$I = \tfrac{1}{2}Mr^2 = \frac{1.90\times10^{-3}}{2}\cdot(0.060\cdot 25.4\times10^{-3})^2 = 2$$

HEX NUTS: $M = 1.60\,gm,\ r \simeq \frac{de}{2} = 0.100\,in$

$$I = Mk_o^2 = 1.60\times10^{-3}\cdot(0.100\cdot 25.4\times10^{-3})^2 = 10$$

AXLE SLEEVE: $M = 0.30\,gm,\ r \simeq \frac{de}{2} = 0.110\,in$

$$I = Mk_o^2 = 0.30\times10^{-3}\cdot(0.110\cdot 25.4\times10^{-3})^2 = 2$$

NO JIVE 3-IN-1
M, I, k_o SUMMARY

COMPONENT	M ($\times 10^{-3}$ kg)	I ($\times 10^{-9}$ kg·m^2)
YO-YO HALVES (2)	49.60	18070
ASSEMBLY PARTS		
AXLE SCREW (1)	1.90	2
HEX NUTS (2)	1.60	10
AXLE SLEEVE (1)	0.30	2
	53.40	18084

$$k_o = \left(\frac{I}{M}\right)^{\frac{1}{2}} = \left(\frac{18084 \times 10^{-9}}{53.4 \cdot 10^{-3}}\right)^{\frac{1}{2}} \cdot \frac{1}{25.4 \times 10^{-3}}$$

$$\underline{\underline{I = 18084 \times 10^{-9}}}\ \text{kg-m}^2$$

$$\underline{\underline{k_o = 0.72}}\ \text{in}$$

ASSEMBLY PARTS IN THIS YO-YO, LIGHT AND CLOSE TO THE AXIS OF ROTATION, CONTRIBUTE LITTLE TO THE MOMENT OF INERTIA. THE WEIGHT OF THESE PARTS MUST NOT BE IGNORED IN DETERMINING THE RADIUS OF GYRATION.

TORNADO 2™
*AREA ELEMENT ANALYSIS

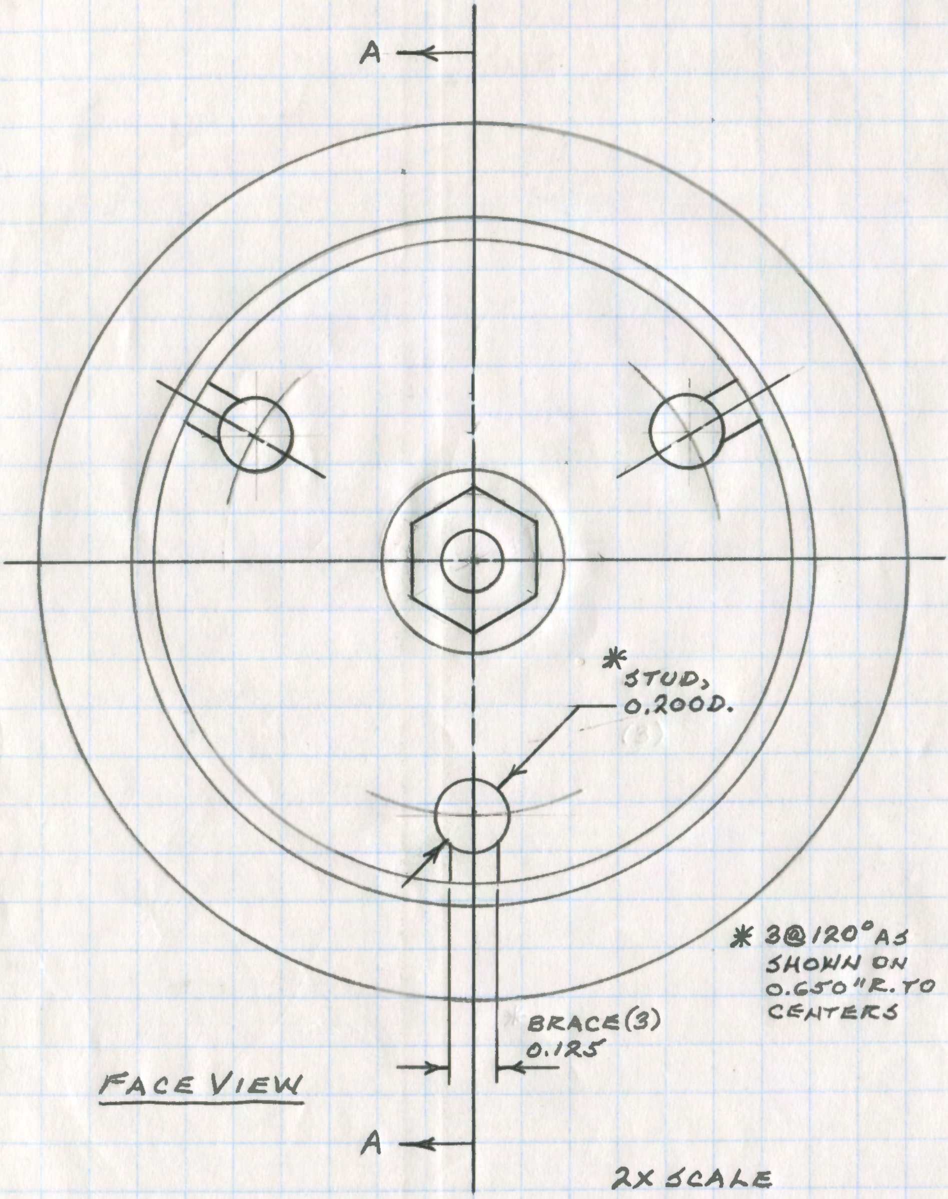
THE CONTEMPORARY BALL BEARING AXLE AND RIM-WEIGHTED TORNADO 2 IS AN EXCELLENT SUBJECT FOR THE EARLIER EXPERIMENT AND FOR THIS ANALYSIS.

THE ACCESSORY FACTORY INSTALLED WEIGHT RINGS AND CARD INSERTS OFFER CONVENIENT QUANTITATIVE EVALUATION OF CHANGES IN WEIGHT AND WEIGHT DISTRIBUTION EFFECTS ON MOMENT OF INERTIA AND RADIUS OF GYRATION, GIVEN:

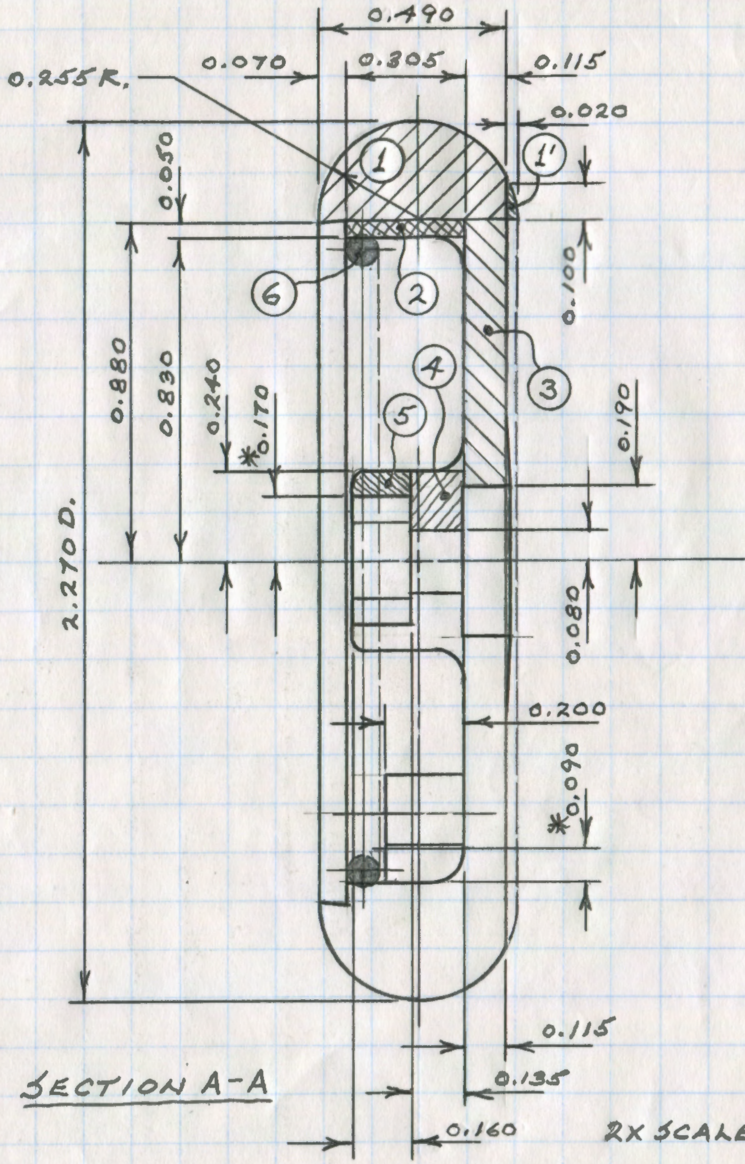
1. PRESENCE OF WEIGHT RINGS AND CARD INSERTS; THE STANDARD CONFIGURATION.
2. ONLY WEIGHT RINGS REMOVED.
3. ONLY CARD INSERTS REMOVED.
4. RINGS AND CARD INSERTS REMOVED.

CASE 1 AND 2 RESULTS FOR I AND k_o ARE DETAILED ON PAGE 32. THESE AND SIMILAR EVALUATIONS, EASILY PERFORMED FOR CASES 3 AND 4, YIELD USEFUL RESULTS AND INSIGHTS — ESPECIALLY IN THE CREATION OF NEW AND INNOVATIVE PRODUCTS, AND IN DESIGN OF MODIFICATIONS TO IMPROVE THE PERFORMANCE OF EXISTING PRODUCTS.

* SEE MONOGRAPH I, "RADIUS OF GYRATION" IN THIS SERIES FOR DETAILED INFORMATION.



TORNADO 2™ - YO-YO HALF



SECTION A-A

2X SCALE

NOTES:

1. DIMENSIONS MARKED* ARE ESTIMATED EQUIVALENTS.
2. SECTION 1' IS RETAINED; FILLETS (2) ARE IGNORED.
3. SECTIONS 1 THRU 5, THE STUDS AND BRACES, ARE PLASTIC; SECTION 6 IS STEEL AND REMOVABLE.

YO-YO HALF PROFILE

YO-YO ELEMENT		VOLUME (in^3)
①+①′	$V=\left(\frac{\pi x^2}{2}\right)\cdot 2\pi\cdot(r+0.424x)$	
	$=\frac{\pi\cdot 0.255^2}{2}\cdot 2\pi\cdot(0.880+(0.424\cdot 0.255))$	$=0.634$
②	$V\overset{*}{=}(x\cdot y)\cdot 2\pi\cdot(r+\frac{y}{2})$	
	$=(0.305\cdot 0.050)\cdot 2\pi\cdot(0.830+\frac{0.050}{2})$	$=0.082$
③	$V\overset{*}{=}(0.115\cdot 0.690)\cdot 2\pi\cdot(0.190+\frac{0.690}{2})$	$=0.267$
④	$V\overset{*}{=}(0.135\cdot 0.160)\cdot 2\pi\cdot(0.080+\frac{0.160}{2})$	$=0.022$
⑤	$V\overset{*}{=}(0.160\cdot 0.070)\cdot 2\pi\cdot(0.170+\frac{0.070}{2})$	$=0.014$
BRACES,	$V=3\cdot \text{AREA}\cdot \text{HEIGHT}$	
	$=3\cdot 0.125\cdot 0.040\cdot 0.200$	$=0.007$
STUDS,	$V=3\cdot \text{AREA}\cdot \text{HEIGHT}$	
	$=3\cdot\frac{\pi\cdot 0.200^2}{4}\cdot 0.200$	$=0.019$

YO-YO HALF, $V=1.045\,in^3$

NOTE: THE YO-YO HALF WEIGHT $M=20.6\,gm$. THUS THE PLASTIC DENSITY $\rho=M/V$; $\rho=19.71\,gm/in^3$.

* DENOTES COMMON EQUATIONS.

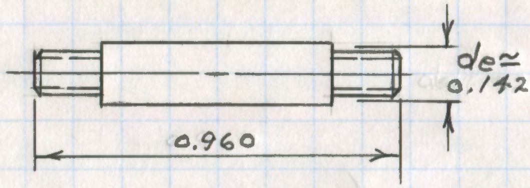
YO-YO HALF PROFILE

YO-YO ELEMENT	MOMENT OF INERTIA	I $(kg\text{-}m^2 \times 10^{-9})$
① + ①′	$I = Mk_o^2 = \rho V(r + 0.424x)^2$	
	$= 19.71 \cdot 0.634\times10^{-3} \cdot ((0.880 + (0.424 \cdot 0.255)) \cdot 25.4\times10^{-3})^2$	
	$I = 7872 \times 10^{-9}\ kg\text{-}m^2$	7872
②	$I \overset{*}{=} \frac{1}{2}M(r_1^2 + r_2^2) = \frac{1}{2}\rho V(r_1^2 + r_2^2)$	
	$= \frac{19.71 \cdot 0.082\times10^{-3}}{2} \cdot (0.830^2 + 0.880^2) \cdot (25.4\times10^{-3})^2 =$	763
③	$I \overset{*}{=} \frac{19.71 \cdot 0.267\times10^{-3}}{2} \cdot (0.190^2 + 0.880^2) \cdot (25.4\times10^{-3})^2 =$	1376
④	$I \overset{*}{=} \frac{19.71 \cdot 0.022\times10^{-3}}{2} \cdot (0.080^2 + 0.240^2) \cdot (25.4\times10^{-3})^2 =$	9
⑤	$I \overset{*}{=} \frac{19.71 \cdot 0.014\times10^{-3}}{2} \cdot (0.170^2 + 0.240^2) \cdot (25.4\times10^{-3})^2 =$	8
BRACES (3):	$I = Mr^2 = \rho V r^2;\ r = 0.785\ in$	
	$= 19.71 \cdot 0.007\times10^{-3} \cdot (0.785 \cdot 25.4\times10^{-3})^2 =$	55
STUDS (3):	$I = Mr^2 = \rho V r^2;\ r = 0.650\ in$	
	$= 19.71 \cdot 0.019\times10^{-3} \cdot (0.650 \cdot 25.4\times10^{-3})^2 =$	102
	YO-YO HALF,	$\underline{\underline{I = 10185}}$ $\times10^{-9}\ kg\text{-}m^2$

⑥ WEIGHT RINGS ARE EVALUATED ON PAGES 30 AND 31 AS ASSEMBLY PARTS.

ASSEMBLY PARTS

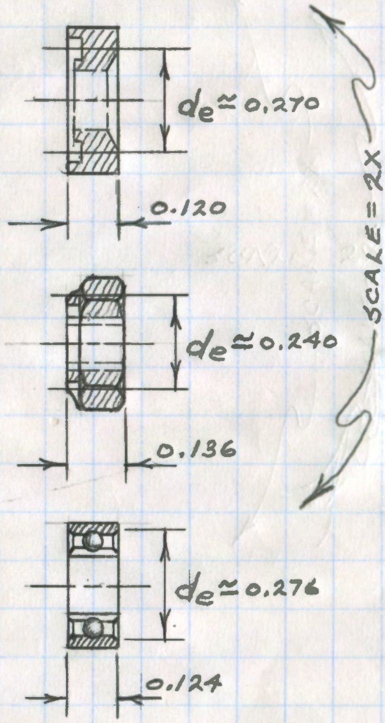
AXLE SCREW (1):
STEEL, 0.156 D.
M = 1.90 gm



SPACERS (2):
STEEL, 0.373 D.
M = 1.15 gm EACH

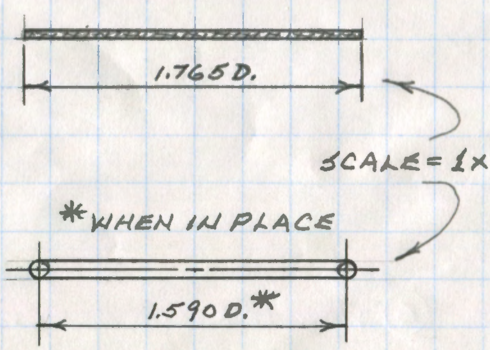
HEX. LOCK NUTS (2):
STEEL, 0.350" @ CRNRS
M = 0.75 gm EACH

BALL BEARING (1):
STEEL, 0.312 D.
M = 0.50 gm



CARD INSERTS (2):
0.037" THICK
M = 1.95 gm EACH

⑥ WEIGHT RINGS (2):
0.090 D. STEEL WIRE
M = 4.1 gm EACH



ASSEMBLY PARTS
MOMENT OF INERTIA

AXLE SCREW (1): $M = 1.90\,gm,\ r \simeq \frac{d_e}{2} = 0.071\,in$

$$J = \frac{1}{2}Mr^2 = \frac{1.90\times10^{-3}}{2}\cdot(0.071\cdot25.4\times10^{-3})^2$$

$$J \simeq 3\times10^{-9}\ kg\text{-}m^2$$

SPACERS (2): $M = 2.30\,gm,\ k_o \simeq \frac{d_e}{2} = 0.135\,in$

$$J = Mk_o^2 = 2.30\times10^{-3}\cdot(0.135\cdot25.4\times10^{-3})^2$$

$$J \simeq 27\times10^{-9}\,kg\text{-}m^2$$

HEX NUTS (2): $M = 1.50\,gm,\ k_o \simeq \frac{d_e}{2} = 0.120\,in$

$$J = Mk_o^2 = 1.50\times10^{-3}\cdot(0.120\cdot25.4\times10^{-3})^2$$

$$J \simeq 14\times10^{-9}\,kg\text{-}m^2$$

BALL BEARING (1): $M = 0.50\,gm,\ k_o \simeq \frac{d_e}{2} = 0.138\,in$

$$J = Mk_o^2 = 0.50\times10^{-3}\cdot(0.138\cdot25.4\times10^{-3})^2$$

$$J \simeq 6\times10^{-9}\,kg\text{-}m^2$$

CARD INSERTS (2): $M = 3.90\,gm,\ r = \frac{D}{2} = 0.882\,in$

$$J = \frac{1}{2}Mr^2 = \frac{3.90\times10^{-3}}{2}\cdot(0.882\cdot25.4\times10^{-3})^2$$

$$J = 979\times10^{-9}\ kg\text{-}m^2$$

WEIGHT RINGS (2): $M = 8.20\,gm,\ r = \frac{D}{2} = 0.795\,in$

$$J = Mk_o^2 = 8.20\times10^{-3}\cdot(0.795\cdot25.4\times10^{-3})^2$$

$$J = 3344\times10^{-9}\,kg\text{-}m^2$$

NOTE: WEIGHT RINGS ARE OPTIONAL; SEE YO-YO HALF SECTION A-A DETAILS AND NOTE 3, PAGE 27.

TORNADO 2 – M, J, k_o SUMMARY

COMPONENT	M ($\times 10^{-3}$ kg)	J ($\times 10^{-9}$ kg-m²)
YO-YO HALVES (2)	41.20	20370
ASSEMBLY PARTS		
AXLE SCREW (1)	1.90	3
SPACERS (2)	2.30	27
HEX NUTS (2)	1.50	14
BALL BEARING (1)	0.50	6
CARD INSERTS (2)	3.90	979
	[1] 51.30	[1] 21399
WEIGHT RINGS (2)	8.20	3344
	[2] 59.50	[2] 24743

$$1.\ k_{o,\text{NO RINGS}} = \left(\frac{J}{M}\right)^{\frac{1}{2}} = \left(\frac{21399}{51.3}\right)^{\frac{1}{2}} \cdot \frac{1}{25.4\times10^{-3}}$$

$$\underline{\underline{k_o = 0.804 \simeq 0.80\,\text{in}}}$$

$$2.\ k_{o,\text{RINGS IN}} = \left(\frac{J}{M}\right)^{\frac{1}{2}} = \left(\frac{24743}{59.5}\right)^{\frac{1}{2}} \cdot \frac{1}{25.4\times10^{-3}}$$

$$\underline{\underline{k_o = 0.803 \simeq 0.80\,\text{in}}}$$

NOTE THAT THE WEIGHT RINGS ADD ALMOST 16% TO THE WEIGHT AND MOMENT OF INERTIA, BUT HAVE NO SIGNIFICANT EFFECT ON THE RADIUS OF GYRATION. k_o FOR THE RINGS IS 0.795 ≃ 0.800 in; WITHOUT RINGS, k_o = 0.804 in.

EXPERIMENT – ANALYSIS CORRELATION

THE ACADEMIC YO-YO EXPERIMENT RELIES ON THE FREE BODY ANALYSIS AND THE CONSTANT $g = 9.81\ m/sec^2$ ($32.2\ ft/sec^2$), ACCELERATION DUE TO GRAVITY. THE AREA ELEMENT ANALYSIS IS ENTIRELY INDEPENDENT OF THE GRAVITY CONSTANT. TARGETING MOMENT OF INERTIA I AND RADIUS OF GYRATION k_o, THE RESULTS OF THE TWO METHODS MUST BE IN REASONABLE AGREEMENT IF THE METHODS ARE VALID.

CORRELATION CHART:

YO-YO	RESULT: EXPERIMENT	RESULT: ANALYSIS
NO JIVE 3-IN-1		
I, $\times 10^{-9}\ kg\text{-}m^2$	17335	18084
k_o, in	0.71	0.72
TORNADO 2 (WITH RINGS)		
I, $\times 10^{-9}\ kg\text{-}m^2$	24306	24743
k_o, in	0.80	0.80

THE CORRELATIONS ARE WELL WITHIN THE ESTIMATED EXPERIMENT "GARBAGE IN, GARBAGE OUT" LIMITS (SEE PAGE 16). ACCURATE MEASUREMENTS AND CALCULATIONS CAN EASILY YIELD ACCEPTABLE CORRELATIONS.

EXPERIMENT TECHNIQUE

EXPERIMENTS, LIKE GOOD PRODUCTS, NEED GOOD DESIGN, AND AS IN PRODUCT MANUFACTURE, NEED CAREFUL EXECUTION AND QUALITY CONTROL. FOR BEST RESULTS:

1. MEASURE d TO THE NEAREST 1/4" OR BETTER.
2. MEASURE M TO THE NEAREST 0.10 gm; SPRING SCALES ARE INADEQUATE.
3. r MUST BE DETERMINED WITH CARE; LEVEL WIND THE FILAMENT IN NEAT AND UNIFORM LAYERS BEFORE MEASURING D_{max}.
4. TIME t ON EACH TRIAL TO THE NEAREST 0.01 sec; ANALOG WATCHES TYPICALLY READ ONLY IN 0.1 sec INCREMENTS.
5. GIVEN VALID AND ACCURATE DATA FOR THE ABOVE VARIABLES, CALCULATION QUALITY IS CONTROLLED BY CHECKING AND RECHECKING.

—

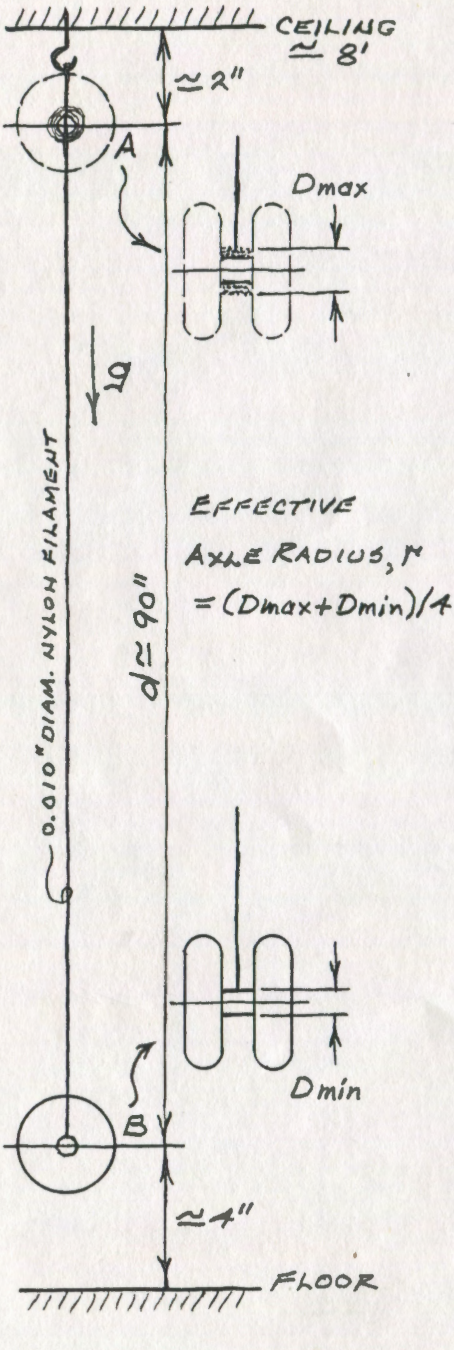
PRACTICE AND PATIENCE ARE NEEDED TO ACHIEVE LEVEL WINDING LAYERS IN 3 ABOVE; GOOD EYESIGHT AND GOOD LIGHTING HELP.

IN 4 ABOVE, REACTION TIME ERRORS IN STARTING AND STOPPING THE WATCH NEED ATTENTION. PRACTICE SOME TIME TRIALS UNTIL READINGS FALL WITHIN A RANGE OF 0.10 sec OR LESS. CONSECUTIVE READINGS IN EACH SET OF SEVEN, WITH EACH READING IN THE 0.10 sec RANGE, CAN YIELD AN ACCURATE AVERAGE VALUE FOR t.

EXPERIMENT
RECORD MASTERS

EXPERIMENTERS: PLEASE FEEL FREE TO COPY THE RECORD MASTERS (SHEETS 1 AND 2) FOR PERSONAL USE ONLY.

NAME ______________________ Yo-Yo ______________________



TRIAL DATA:

$g = 9.81\ m/sec^2$

$d =$ ______ in

$M =$ ______ gm

$D_{max} =$ ______ in

$D_{min} =$ ______ in

$r =$ ______ in

TRIAL TIME:

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______

TOTAL = ______ sec

$t = TOTAL/5$

$t =$ ______ sec

CALCULATION WORKSHEET

UNIT CONVERSIONS:

$g = 9.81 \ m/sec^2$

$t =$ ______ sec

$d =$ ______ $in \cdot 25.4 \times 10^{-3} m/in =$ ______ m

$M =$ ______ $gm \times 10^{-3} kg/gm =$ ______ $\times 10^{-3} kg$

$r =$ ______ $in \cdot 25.4 \times 10^{-3} m/in =$ ______ $\times 10^{-3} m$

MOMENT OF INERTIA, J $kg\text{-}m^2$:

$$J = \left(\frac{gt^2}{2d} - 1\right) M r^2 \ kg\text{-}m^2$$

$$J = \left(\frac{9.81 \frac{\cancel{m}}{\cancel{sec^2}} \cdot \qquad \cancel{sec^2}}{2 \cdot \qquad \cancel{m}} - 1\right) \cdot \qquad \times 10^{-3} kg \cdot \left(\qquad \times 10^{-3} m\right)^2$$

$$\underline{\underline{J = \qquad \times 10^{-9}}} \ kg\text{-}m^2$$

RADIUS OF GYRATION, k_o in:

$$k_o = \left(\frac{J}{M}\right)^{\frac{1}{2}} m$$

$$k_o = \left(\frac{\qquad \times 10^{-9} \cancel{kg}\text{-}m^{\cancel{2}}}{\qquad \times 10^{-3} \cancel{kg}}\right)^{\frac{1}{2}} / 25.4 \times 10^{-3} \frac{\cancel{m}}{in}$$

$$\underline{\underline{k_o = \qquad}} \ in$$

DATE ______ BY ________

AUTHOR'S NOTE

MOMENT OF INERTIA AND RADIUS OF GYRATION, BUILT-IN DETERMINANTS OF PERFORMANCE CHARACTERISTICS, ARE THOROUGHLY TREATED IN MONOGRAPHS I AND II. FUTURE ENTRIES IN THE SERIES WILL MENTION AND USE THEM, BUT INVESTIGATE THEM FURTHER ONLY IN THE SPECIAL CASE OF "BUTTERFLY" STYLE YO-YOS. IN ALL YO-YO DESIGNS WEIGHT, WITH ITS RADIAL AND LATERAL DISTRIBUTION, IS A MOST IMPORTANT CONSIDERATION - NOT TO BE IGNORED. SPIN DURATION, FIXED AXLE AND TRANSAXLE PERFORMANCE, STRING AND STRING GAP EFFECTS, AND GYROSCOPIC CHARACTERISTICS ARE SOME INTERESTING SUBJECTS PLANNED FOR FUTURE INVESTIGATION.

READERS ARE INVITED TO EVALUATE AND COMMENT; ESPECIALLY REGARDING ERRORS DETECTED. BE AWARE THAT ALL PRODUCTS ARE SUBJECT TO TOLERANCE EFFECTS, DESIGN CHANGES, AND OTHER DIFFERENCES PRODUCING DEVIATION FROM INPUT DATA AND RESULTS GIVEN HERE.

HAPPY DAYS-

Don Watson

Captain Yo

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4. SIX EASY PIECES, RICHARD P. FEYNMAN, (SOFTCOVER) 1994, PERSEUS BOOKS, READING MA, PG. 24.

5. "SURELY YOU'RE JOKING, MR. FEYNMAN!", RICHARD P. FEYNMAN, (SOFTCOVER) 1989, BANTAM BOOKS, NEW YORK NY.

* * *

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